Dear Accelerated Pre-Calculus Student,

I am excited to have you enrolled in our class for next year! We will learn a lot of material and do so in a fairly short amount of time. This class will be intellectually challenging, and hopefully, very rewarding. Because of the amount of material that we need to cover in this course, we have put together a summer assignment that we feel you need to have attempted by the beginning of the school year. My best advice is to wait until August to complete the work, that way the material is fresh in your mind. We will spend a few days at the beginning of the year reviewing the material, however, you will need to have complete the majority of this packet by the time school begins next fall.

Sincerely,
Mr. Jost

Here are the parts of the assignment to complete:

1) Complete problems 1-65 on pages 3-12. You are encouraged to use the space provided.

2) Read through pages 13-15, a tutorial on solving equations and inequalities graphically.

3) Complete problems 1-12 on page 16 on separate paper.

1) Find the slope of the line through the points (-1, -2) and (4, -5).

2) Find an equation in point-slope form for the line through the point (2, -1) and slope $m = -2/3$.

3) Rewrite the equation $3x - 4y = 7$ in slope-intercept form.

In questions 4 – 8, find the equation of the line in slope-intercept form:

4) The line through (3, -2) with slope $m = 4/5$

5) The line passing through the points (-1, -4) and (3, 2)

6) The line through (-2, 4) with slope $m = 0$.

7) The line through (2, -3) and parallel to the line $2x + 5y = 3$.

8) The line through (2, -3) and perpendicular to the line $2x + 5y = 3$.

In questions 9 – 13, use properties of exponents to simplify the expressions. Your answers should contain positive exponents only.

9) $\dfrac{(uv^2)^3}{v^2u^3}$

10) $(3x^2y^3)^{-2}$

11) $\left(\dfrac{-2x^3y}{5x^7}\right)^2$

12) $(x^{-3}y^{-1})^{-1}(x^{-3}y^0)^2$

13) $\left(\dfrac{s^{-3}}{4t}\right)^{-3}\left(\dfrac{5t}{s^{-7}}\right)^{-2}$

For questions 14 – 17, simplify the rational expressions:

14) $\dfrac{12x}{15} \bullet \dfrac{10x}{21}$

____

15) $\dfrac{9xy}{12xy^2} \div \dfrac{6y^3}{21x^3y}$

____

16) $\dfrac{x^2+x-2}{x^2-4x-12} \bullet \dfrac{x^2-5x-6}{x^2-2x+1}$

____

17) $\dfrac{2x^2+7x-4}{x^2-12x+20} \div \dfrac{2x^2-x}{2x^3-24x^2+40x}$

____

18) Use the quadratic formula to solve the equation $3x^2+4x-1=0$

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In exercises 19- 33, solve each equation algebraically. (Quadratics that can be solved by factoring should be solved by factoring.)

19) 3x – 4 = 6x + 5

____

20) (5 - 2y) – 3(1 - y) = y + 1

____

21) x(2x + 5) = 4(x + 7)

____

22) $|4x+1|=3$

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23) $\frac{x-2}{3}+\frac{x+5}{2}=\frac{1}{3}$

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24) $\frac{x}{x-3}-\frac{7}{x+5}=\frac{24}{x^2+2x-15}$

____

25) $\frac{4}{x-1}=\frac{x+1}{12}$

____

26) $\frac{2x-3}{x+1}=1$

____

27.) $\frac{3x}{x+1}+\frac{5}{x-2}=\frac{15}{x^2-x-2}$

____

28) $\dfrac{3}{x} - x = 2$

29) $16x^2 - 24x + 7 = 0$

30) $6x^2 + 7x = 3$

31) $4x^2 - 20x + 25 = 0$

32) $-9x^2 + 12x - 4 = 0$

33) $3(3x-1)^2 = 21$

Solve the cubic equations by factoring:

34) $3x^3 - 19x^2 - 14x = 0$

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35) $x^3 + 2x^2 - 4x - 8 = 0$

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For questions 36 – 40, solve the inequality algebraically. Express your answer in interval notation.

36) $|3x + 2| \leq 4$

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37) $|2x + 1| > -5$

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38) $5x + 1 \geq 2x - 4$

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39) $|5x - 2| > 3$

____

40) $\frac{3x - 5}{4} \leq -1$

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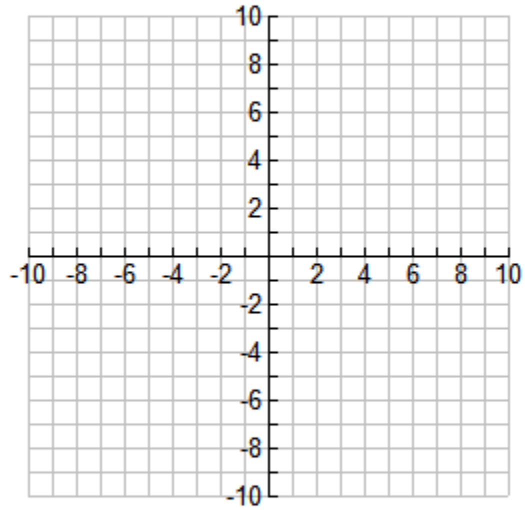
For the given rational function, find the items listed. Sketch an accurate graph of the function.

41) $y = \dfrac{9x^2 - 25}{3x^2 - 7x - 20}$



Work for factoring:

Domain: ______________________

Discontinuities

Holes: ______________

Vertical Asymptote(s): ________________

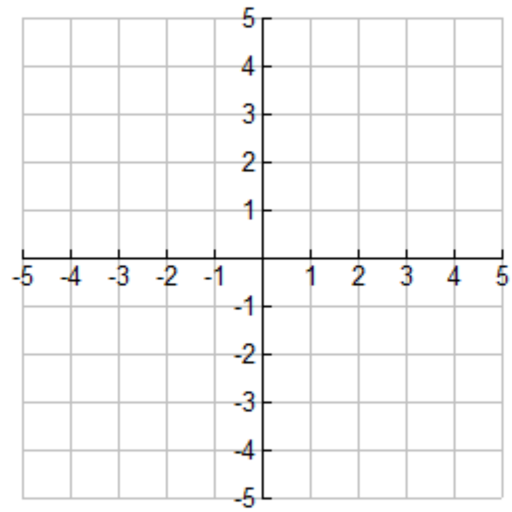
End Behavior (Horizontal Asymptote): ___________

X-intercepts:

Y-intercept:

42) $y = \dfrac{2x^2 + 9x - 5}{x^2 - 2x - 15}$

Work for factoring:



Domain: ______________________

Discontinuities

Holes: ______________

Vertical Asymptote(s): ________________

End Behavior (Horizontal Asymptote): __________

X-intercepts:

Y-intercept:

For questions 43-50, evaluate the logarithmic expression without using a calculator:

43. $\log 100$

44. $\log_3 81$

45. $\log_2 \sqrt{8}$

46. $\log 1$

47. $\log_6 1$

48. $\log_2 \frac{1}{8}$

49. $\log \frac{1}{1000}$

50. $5^{\log_5 8}$

For questions 51-53, **Use the properties of logarithms to write the expression as a sum or difference of logarithms or multiples of logarithms. Simplify if possible.**

51. $\log(xy^3)$

52. $\log \frac{x}{1000}$

53. $\log_2 \left(8xy^4\right)$

For questions 54-56, Use the properties of logarithms to write the expression as a single logarithm.

54. $3\log(x^3 y) + 2\log(yz^2)$

55. $\frac{1}{2}\log x - 3\log y + \log z$

56. $2\log x + 3\log y$

For questions 57-65, solve the exponential/logarithmic equation. Leave your answer in exact form.

57. $3\left(4^{x/2}\right) = 96$

58. $\log_4(x-5) = -1$

59. $\log x = 4$

60. $\log(x-3) + \log(x+4) = 3\log 2$

61. $2\log_3(x) - 3 = 0$

62. $\log(x) - \log(4) = 2$

63. $5^x = 10$

64. $3^{x-2} = 7$

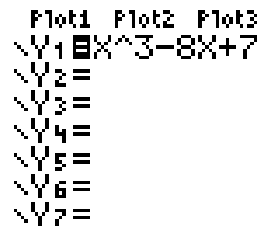
65. $2 - 3(4^x) = -8$

Part I: How to Solve Equations Graphically

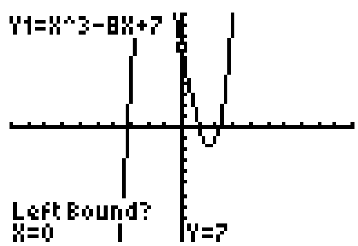
To solve an equation graphically, you can use the zero method or the intersect method. (EITHER METHOD WORKS FOR ANY EQUATION)

Example 1: Solve the equation $x^3 - 8x + 9 = 2$, using the zero method.

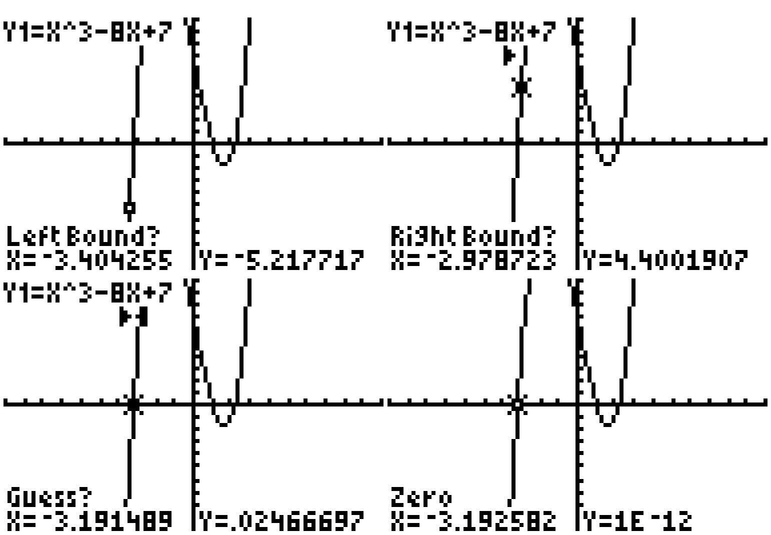
To use the zero method, you'll have to set the equation equal to zero, before you enter the equation into Y= on your calculator. If $x^3 - 8x + 9 = 2$, then $x^3 - 8x + 9 = 0$. This is the equation we insert into Y=.



Then, go the graph screen. Hit 2nd trace and select option 2 (zero).



Next, move your cursor to the leftmost zero, and use the left arrow to go left (it makes no difference how far left) of the leftmost zero. (In this case, that would be below the leftmost zero). Hit enter. Then, use the right arrow to go right (it makes no difference how far right) of the leftmost zero. (In this case that would be above the rightmost zero). Hit enter. Then, it will ask you for a guess, and you'll move your cursor close to the actual zero and hit enter again. Your calculator will display the answers. Your successive steps are shown in the captures below.

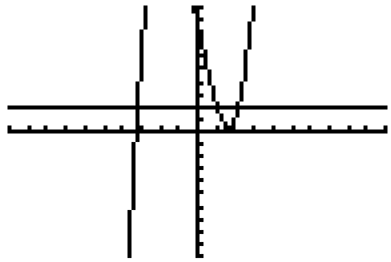


The zero occurs at x = -3.193 (round to three decimal places). Note: my calculator says that the y-value for this x-coordinate is 1E-12 when it should say that the y-coordinate is 0. However, 1E-12 is simply the scientific notation for the number 1×10^{-12}, which is .000000000001 practically zero.

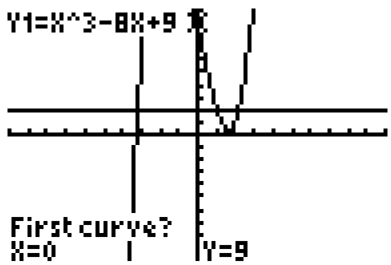
You will then find the remaining zeros of this function using the same method.

Example 2: Solve the same equation as in example 1, but use the intersect method this time.

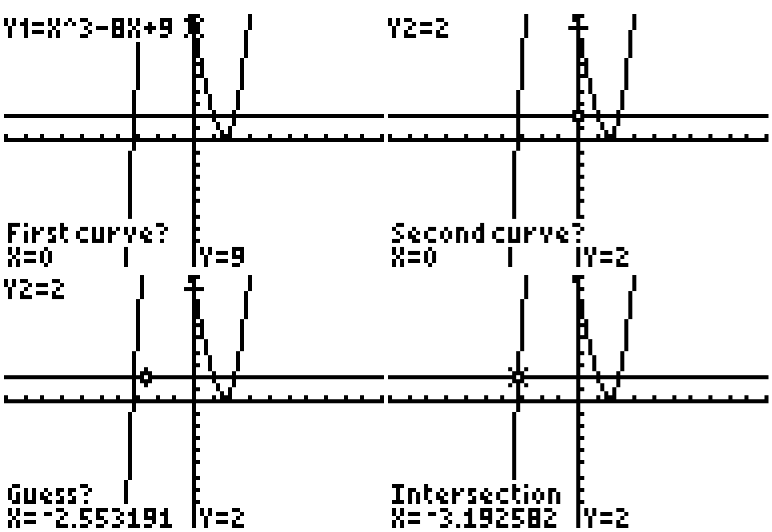
Since we are given $x^3 - 8x + 9 = 2$, and are told to use the intersect method, we'll input the left-hand side of the equation into Y_1, and the right-hand side of the equation into Y_2.



Next, go hit 2nd, TRACE, and choose the intersect option. Your screen will look like this:



Notice one huge difference here. Your calculator is asking you for the first curve, the second curve, and your guess instead of the left bound, right bound, and your guess. Look in the upper left-hand corner of your screen at this point and notice that it says $Y_1 = x^3 - 8x + 9$. Once you hit enter, your calculator will now consider this to be one of the curves that you'll be intersecting. Then, your calculator will ask you "second curve?" The good news here is that the calculator will automatically move on to whatever you have enter in Y_2. So, just hit enter again. Then, when the calculator asks for your guess, you need to move your cursor to the intersection point of interest. We have to tell the calculator which intersection point we want in this case, especially because we have three points to choose from!



The intersection occurs at x = -3.193 (round to three decimal places). Note: my calculator says that the y-value for this x-coordinate is 2. But, this isn't a solution to our equation. From the equation given, it is apparent that we our only concerned about solutions for x that make the equations equal.

You would then use the same process to find the other two intersection points, whose x-values will yield the other two solutions.

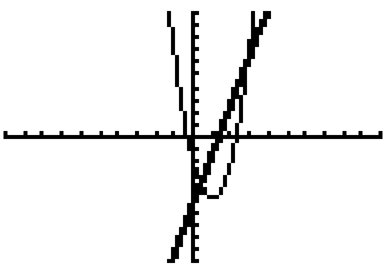
Part II: Solving Inequalities Graphically

For the remainder of your mathematical career, it will be beneficial for us to be able to solve inequalities such as $3x^2 - 6x - 2 \geq 4x - 5$. Unfortunately, we haven't yet discussed the algebraic way to solve these inequalities, which would be done via a sign chart. So, for now, we'll settle on solving them graphically.

Before we even go to a graph, we need to figure out what it is that we're looking for. In the statement, $3x^2 - 6x - 2 \geq 4x - 5$, we are being asked the question "give the x-values where the y-values of the parabola are greater than or equal to the y-values of the line?" This would be equivalent to asking "for what x-values is the parabola either higher than the line or at the same height as the line?" The key to answering this question will be knowing the intersection points.

Example 1: Solve $3x^2 - 6x - 2 \geq 4x - 5$.

First, go to your calculator, and use the intersection method (personally, I prefer the intersection method to the zero method as I think it's easier – but, if you would rather get everything to one side and use the zero method – feel free). The intersection method will tell us that these curves intersect at $x = \frac{1}{3}, x = 3$. I don't even need to mention the y-values here because they will be of no consequence in our final solution.

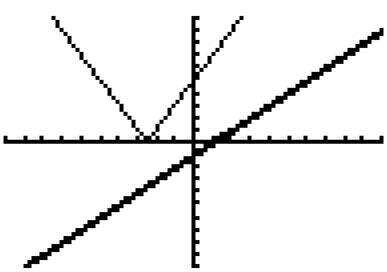


Comparing the graphs, we see that the parabola is always higher or at the same y-value as the line left of the first intersection point and right of the second intersection point.

Thus, our solution is $\left(-\infty, \frac{1}{3}\right] \cup [3, \infty)$.

If this confuses you, take a straightedge and hold it vertically at random points on the graph. Then, ask yourself, what's higher here: the parabola or the line? If the parabola is higher or at the same level than the line at that particular x-coordinate, than this x-coordinate will be part of the solution to your inequality.

Example 2: Solve $|2x + 5| > x - 1$.



This question is asking us where the absolute value function is higher than the line. Notice that this question is not concerned with where the functions are at the same level, as the problem contains a greater than sign and not a greater than or equal to sign. Anyway, since the absolute value function never intersects the line, this question is actually pretty easy. The absolute value function is always higher than the line. Thus, the inequality is true for all real numbers of x. Answer: $(-\infty, \infty)$

Practice Problems

Directions: Solve the following problems using the methods indicated. On your paper, make a rough sketch of the graph, and label the zeros/intersection points. If necessary, round solution(s) to three decimal places.

On some questions, you may need to alter you viewing window to see a better picture of the graph!

Solve the following equations using the zero method:

1) $x^2 - 4x - 11 = 2$

2) $\sqrt{3x+5} = 1$

3) $x^3 + x^2 - 8x + 5 = 0$

Solve the following equations using the intersect method:

4) $2x + 3 = |x - 1|$

5) $x^3 - 2x + 4 = \sqrt{3x+7}$

6) $2x + 3 = x^2 - 4x - 1$

Solve the following inequalities:

7) $|3x+1| > x - 4$

8) $x^2 + 3x - 4 \le x^2$

9) $3x - 2 > 4x + 1$

10) $|3x+2| \le -3$

11) $|3x+2| > -3$

12) $x^3 - 3x + 1 \le 2x^2 + 4$